HW4B. Written Homework 4B. Power Series Representations. **Due Week 4 Friday 11:59PM**

Name:

Instructions: Upload a pdf of your submission to **Gradescope**. This worksheet is worth 20 points: up to 8 points will be awarded for accuracy of certain parts (to be determined after the due date) and up to 12 points will be awarded for completion of parts not graded by accuracy.

(1) For each of the given power series, find the radius of convergence R and the interval of convergence I. You may use any applicable convergence test covered in this course.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{n+1}}(x-3)^n$ **(b)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{2n+1}x^{2n+1}$ **(c)** $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

(2) Let $f(x) = \sin(x^2)$.

(a) Let $g(x) = \sin(x)$ and let $T_n(x)$ be the n^{th} order Taylor polynomial of $g(x)$ about $x = 0$. Let $R_n(x) = g(x) - T_n(x)$, the n^{th} order remainder of $T_n(x)$ for all $n \geq 0$.

Using Taylor's Inequality, find all $x \in \mathbb{R}$ such that $\lim_{n\to\infty} R_n(x) = 0$.

(b) Find a power series representation for $g(x)$ and identify its radius of convergence R_g.

(c) Determine a power series representation for $f(x)$ and identify its radius of convergence R_f.

(d) Approximate $\displaystyle\int_0^{0.5} \sin(x^2)\,dx$ to 6 decimal places. Justify why your approximation is valid.